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### Search for a New Conceptual Bookkeeping Model

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# **SEARCH FOR A NEW CONCEPTUAL BOOKKEEPING MODEL:**

## **DIFFERENT LEVELS OF ABSTRACTION**

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### **Abstract**

Nowadays, every bookkeeping system used in practice is automated. Most bookkeeping software and integrated information systems are based on databases. In this paper, we develop a new conceptual bookkeeping model which is not based on manual techniques, but which is applicable in a database environment. The model is designed as a composition of equations. The starting-point of these equations is the well-known accounting equation. In the development of the model, several levels of abstraction are distinguished: from an abstract level to a more concrete level. Every level of abstraction is described by one equation. This equation has both an input-function and an output-function. With the development of this model, the gap between the bookkeeping literature and bookkeeping practice has been reduced.

**Keywords:** bookkeeping, accounting information systems, conceptual modeling

**JEL classification:** M40, M41

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## 1. Introduction

Bookkeeping as a discipline has a long history. The roots of the present double entry form are found in Pacioli's 1494 work "Summa de Arithmetica, Geometria, Proportioni et Proportionalita". Beek, van Duin, and Vaassen (1997, p.5) describe bookkeeping as follows: "the object of bookkeeping is providing users, inside and outside the organisation, with financial information by recording and processing financial data about the business processes, assets and liabilities of an organisation". In this definition, bookkeeping concerns the whole process from input of data up to the output of financial statements. This bookkeeping definition is typical of the Dutch literature (Bes, 1908; van der Boom et al., 1995; Van Essen, 1998; Groot, 1943; Huijsman, 1924; Kevelam et al., 1990; Kreukniet, 1949; van Liempt et al., 1990; Volmer, 1994; Van Voorene, 1992).

In the Anglo-Saxon literature, Boynton (1955) distinguishes three activities: recordkeeping, bookkeeping, and accounting. So recordkeeping is the recording of financial data in one special component of the bookkeeping system. Recordkeeping contains all recording activities "up to the trial balance". Bookkeeping, on the other hand, implies compiling the financial statements: the balance sheet, income statement or profit and loss statement, and the cash flow statement, and covering the different elements of the bookkeeping system. The bookkeeping procedures usually end when the basic data have been entered into the books of account and the accuracy of each entry has been tested. At that stage, the accounting function takes over. Accounting tends to be used as a generic term covering almost anything to do with the collection and use of basic financial data (Dyson, 1997, p. 11).

In both the Dutch and the Anglo-Saxon literature (Needles and Powers, 1998; Dyson, 1997; Weetman, 1996; Lerner, 1978; Magee, 1972; Whitehead, 1991; Randall and Beckwith, 1989; Glautier and Underdown, Clark, 1978; Gray et al., 1996; Meigs and Meigs, 1992), bookkeeping is discussed using a manual bookkeeping system. This manual system was put into practice many years ago and is based on the traditional accounting cycle, which works with manual techniques. The theories of accounts explaining the rules of bookkeeping also use the traditional bookkeeping systems, by formulating the rules using manual techniques.

As a result of the development of the technology, the manual bookkeeping system has been more and more automated and in almost every organisation the bookkeeping system at this moment is fully automated (Wilson and Sangster, 1992). Most of these automated bookkeeping systems are based on one or more databases (Harper, 1985; Walker and Denna, 1997). Now, there is a gap between the theory of bookkeeping and the system used in bookkeeping practice. In this paper, we reduce this gap by developing a new conceptual bookkeeping model that does not use traditional techniques (Bak, 1985). We distinguish six abstraction levels in this new model. We start with the most abstract level (level 0) which describes only the working of the model, but it does not deliver output in money terms. The next abstraction levels, level 1 up to and including level 3, have each extra input comparing the foregoing levels. Each level has, as a result, more output than the foregoing levels. At level 3 the bookkeeping model can function, and it delivers balance sheets and profit and loss statements over every desired period. These statements deliver no detail information about the composition of wealth or profit. The output of level 4 is more in detail and it fulfils the requirements of legislation and regulation. Level 5 and 6 deliver moreover management information, both financial and quantitative.

In section 2, we discuss the conceptual bookkeeping model in the present literature. In section 3, a new conceptual bookkeeping model is developed. In section 4, six abstraction levels are distinguished; every level is described by an equation. In section 5, conclusions are drawn and suggestions are made for further

research.

## **2. Description of the conceptual bookkeeping model in the existing state of the literature**

### *2.1. Introduction*

A conceptual bookkeeping model describes the bookkeeping process from input up to output. The financial data from the business processes of the organisation are input elements for a bookkeeping system. The data from the business processes can be non-financial, but in that case these data are no direct input. The way in which input of financial data is provided to the bookkeeping system is externally determined. The traditional bookkeeping model subsequently processes these data. The functioning of the traditional bookkeeping model is described in the next subsection. After processing financial data, the bookkeeping model generates output. This output refers to financial information of the preceding period: a balance sheet and a profit and loss statement but also operational information from subsidiary ledgers. This output has been required for every organisation for a long time now. In the next subsections, the existing situation of bookkeeping in literature and practice is described.

### *2.2. The traditional (manual) accounting cycle*

The accounting cycle describes the stages of the processing of financial data. In a manual bookkeeping system, it comprises the following manual procedures (Meigs and Meigs, 1992). Financial data from different business processes are recorded first in special journals. Financial transaction data are collected, sorted per type of transaction, and recorded in the special journal. At the end of a given period, a collective journal entry is made of each special journal. The General Journal contains both the totals (debit and credit) of the balance sheet and the (collective) journal entries of each special journal. The journal entry mentions the general ledger accounts that have to be debited and credited. The total debit amount equals the total credit amount in the journal entry. As a result, the sum of the debit amounts equals the sum of the credit amounts in the general journal: the general journal balance. In a manual bookkeeping system, this is an important check on correctness.

The amounts of the general journal are posted in the ledger accounts in the general ledger at the end of the period. Making collective journal entries requires collective ledger accounts. As a result, information about business operations disappears, for example information about individual creditors. To avoid this loss of information, individual data about the transaction is recorded in a subsidiary ledger. The recording in the special journal and that in the subsidiary ledger take place separately and independently; both are batch processes. Another check is possible, because of the fact that different people perform these recording activities. The total amount of the individual account balances in the subsidiary ledger has to be equal to the balance of the general ledger account.

At the end of a period, say a month, when the journal entries have been posted in the general ledger, the trial balance can be composed. The total of debit amounts and the total credit amount in the general ledger are totalled and have to be equal. The totals of the columns in the trial balance have to equal the general journal's totals. After the trial balance has been compiled, the profit and loss statement and balance sheet can be

formed by putting the balances of all general ledger accounts from the trial balance into the balance sheet or profit and loss statement. These two statements have been prepared, the traditional accounting cycle is complete. This cycle has arisen from a manual bookkeeping practice with practical requirements. In the literature (van Liempt, et al., 1990; Kevelam et al., 1990; Van Essen, 1998) this accounting cycle is still used, however, as part of a conceptual model. In this paper, a conceptual bookkeeping model is defined as the whole of the accounting cycle and the theory of accounts.

### *2.3. Double-entry bookkeeping*

In both the Dutch and the Anglo-Saxon literature, the manual system of bookkeeping discussed is always based on a double-entry system. Double entry has different meanings in the literature. In the Dutch literature (Wagensveld, 1995), double entry means that equity is recorded in both its extent and composition (Van Liempt, et al., 1990). The composition of equity is defined by the components: assets and liabilities. These are stock values: they specify equity at a particular moment. The changes in these stock values are recorded in stock accounts. In addition to these changes in composition, also the changes in the extent of equity are recorded in accounts, describing the causes of the changes in equity. These accounts are flow accounts (Parkinson, 1987). The changes in the extent of equity are specified in income and expenses (Bindenga, 1984).

Corresponding with the explanation above, Olders (1995) gives another explanation of double entry. Olders distinguishes two dimensions of equity, wealth and income. The balance sheet represents wealth (the first dimension), and the profit and loss statement represents income (the second dimension).

In the Anglo-Saxon literature (Avkiran and Scorgie, 1990; Scorgie, 1989; Ijiri, 1982, 1986), the explanation of “double” in double entry is based on the recording technique: every transaction is recorded twice, once debit and once credit. The total debit amount equals the total credit amount. As a result, all financial statements balance.

In the earlier literature, a single-entry bookkeeping system is also discussed (Kreukniet, 1930; Bouman, 1935). In a single-entry bookkeeping system, only the composition of equity is recorded. The extent of equity has to be found by subtracting liabilities from assets.

According to Olders, a single bookkeeping system has only one dimension: wealth. The output of this system is the balance sheet; a profit and loss statement does not exist.

### *2.4. Theories of accounts*

Theories of accounts, or bookkeeping theories, seek to explain the singular mechanism of double-entry bookkeeping, especially the real nature of the different accounts and of their functional and causal relationship. The bookkeeping theories search for the logic of the system and the used theory of transforming economic facts, conditions, and transactions into schemes (Käfer, 1966, p.2). Bookkeeping theories have two purposes; the first is didactical. It has the intention to give student and practitioner a justification for the different rules of double-entry bookkeeping. This justification should help them to understand the bookkeeping process. The second purpose is a scientific one: bookkeeping theories search for a logical foundation and demonstrable construction of a theory of double-entry bookkeeping (McMillan, 1998).

In the literature, different theories of accounts are discussed (Ten Have, 1973). All theories discussed are based on a manual system, because all these theories are outdated. Before 1900, the Personalistic theory was developed (Käfer, 1966; Kreukniet, 1949). The Personalistic theory is based on the idea that two persons are involved with every transaction: receiver and payer. The account receivable is debited and the account payable is credited.

Another theory is the Entity Theory (Van der Boom and Van der Grift, 1981). This theory explains the bookkeeping rules under the assumption that the company is a separate economic unit. Bookkeeping is the justification of all positive and negative values of the entity (assets and liabilities respectively). “Accounts of assets” are debited for increase and credited for decrease of the assets of the firm. “Accounts of liabilities” are debited for decrease and credited for increase of the liabilities of the firm. (Van Voorene, 1993, p. 135; Van der Boom and Van der Grift, 1985, 1986; Vernooij, 1986).

Van Voorene (1993) also discusses the Two-series-of-accounts theory. This theory is about two categories of accounts:

- equity accounts;
- components of equity accounts: assets and liabilities.

If equity increases the equity accounts are credited and the assets and liabilities accounts are debited. If equity decreases the equity accounts are debited and the assets and liabilities accounts are credited.

In most of the Anglo-Saxon literature (for example, Lerner, 1978; Needles and Powers, 1998), the commonly used theory of accounts is the accounting equation:

$$\text{Assets (A)} = \text{Equity (E)} + \text{Liability (L)}. \quad (1)$$

The explanation for the rules of bookkeeping is based on the fact that the accounting equation has to be balanced. The form of the accounting equation is based on the balance sheet.

Although in bookkeeping practice manual techniques are no longer used (Beek and Jager, 1997; Starreveld et al., 1993), they are, however, still mentioned in the bookkeeping literature. Especially the conceptual bookkeeping model is purely based on a manual system; the Dutch theories of accounts confirm this. On the other hand, the Anglo-Saxon theory of accounts, which makes use of the accounting equation, has dropped the manual system.

In this paper, a new conceptual bookkeeping model is developed to replace the traditional (manual) bookkeeping model. In order to disconnect the conceptual model and the manual system, we use the accounting equation as a starting-point. This equation is suitable as a supporting theory of accounts.

### **§ 3 Development of the conceptual bookkeeping model**

#### *§ 3.1 Introduction*

In this section a new conceptual model for bookkeeping is developed. It is especially the bookkeeping model, which describes the process from input to output that causes the discrepancy between literature and practice.

This is the reason why in this section a new conceptual model is developed, uncoupled from manual techniques.

The newly developed model describes equity extensively as wealth, not only wealth accumulated over the past, but also the future wealth of an organisation (Ijiri, 1982). Because the recording of the wealth of an organisation is the central object of the model, transactions from the business processes influencing wealth are the input of the bookkeeping model. The influence of transactions on wealth must be expressed in financial terms (cf. the measurement principle). In other words, financial transactions constitute the input, whereas other, non-financial data attributes of these transactions, can also be taken into account (can be recorded in the model).

The model will be a system of equations. The starting-point is the accounting equation. By defining the components of the accounting equation more detailed, as is done in the conceptual framework, the possibility is created of processing not only transactions from the past but also future (prospective) transactions in the model. Thus the possibility of generating prospective information is created. We still adhere to the “double” of a double entry bookkeeping system. For the model, “double” means the security of equilibrium, and the possibilities of control.

First in section 3.2 a description is given of an autonomously functioning conceptual balance sheet model. To describe this balance sheet model, the single-entry bookkeeping system is chosen as a starting-point. In this bookkeeping system, only the composition of equity is recorded and the extent is determined by balancing.

In section 3.3, an autonomously functioning profit and loss statement model is described. This model is also constructed within the single-entry bookkeeping system. The composition of profit is recorded in its specific components and the extent of profit is determined by balancing.

Then, in section 3.4, the balance sheet model and the profit and loss statement model will be integrated to form an integrated bookkeeping model that generates the balance sheet, and the profit and loss statement. By integrating the two models, the composition of both equity and profit are recorded. The recording of the composition of profit implies the recording of the extent of equity. Therefore, the integrated model is based on the double-entry bookkeeping system.

In section 3.5, an example of the functioning of the integrated model is given on the basis of transactions taking place in a specific organisation.

In section 3.6, the integrated model is expanded with contributed capital and withdrawals by owners.

### *3.2. Balance sheet model*

The description of the balance sheet model focuses on equity, because of the fact that equity measures the performance of the organisation, reflected in transaction data from the business processes<sup>2</sup>. The recording of the composition of equity in this model is continuous. The extent of equity is calculated by summation. Equity is defined as in (1):  $Eq = A - L$ , where,  $Eq$  = Equity,  $A$  = Assets, and  $L$  = Liabilities. Within this conceptual framework Assets ( $A$ ) are defined as “resources controlled by the enterprise as a result of past events, and from which future economic benefits are expected to flow to the enterprise”. Liabilities ( $L$ ) are

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<sup>2</sup> Contributed capital and withdrawals have not yet been taken into account. In section 3.6 the influence of contributed capital and withdrawals will be examined.

defined as “present obligations of the enterprise arising from past events, the settlement of which is expected to result in an outflow from the enterprise of resources embodying economic benefits”. Equity (Eq) is defined as “the residual interest in the assets of the enterprise after deduction of all its liabilities” (International Accounting Standards Committee, 1991).

Based on (1), the total amount of A and the total amount of L determine the value of Eq. The composition of Eq is constituted by A and L. Equation (1) can be written as follows:

$$A = L + Eq . \quad (2)$$

This is the accounting equation that underlies the composition of the balance sheet. Another way to formulate this equation is:

$$A - L - Eq = 0 . \quad (3)$$

The balance sheet is a statement of A, L and Eq at particular points in time. A, L, and Eq have a certain stock value at every point in time. The position of a stock value X up to and including transaction i, is defined as  $X_i$ , for example,  $A_i$  is the stock value of assets up to and including transaction i.

The business processes in an organisation are the source of all kinds of financial transactions, which affect the stock values. The change in a stock value X as the result of transaction i, is defined as  $\Delta X_i = X_i - X_{i-1}$ ; for example, the change in A as a result of transaction i is  $\Delta A_i = A_i - A_{i-1}$ . Despite the changes caused by financial transactions, (2) remains balanced, and (3) is complied with. The reason is that for every financial transaction i ( $i = 1, 2, 3, \dots$ ), the following holds:

$$\Delta A_i = \Delta L_i + \Delta Eq_i \text{ or } \Delta A_i - \Delta L_i - \Delta Eq_i = 0 . \quad (4)$$

In (4) every transaction i generates *exactly* two changes in stock values. For mathematical reasons, it is assumed that all stock values change. The change can be positive, negative, or equal to zero. At the starting moment of any organisation, the assumption is that the stock values, A, L, and Eq, are equal to zero:

$$A_0 = 0, L_0 = 0, \text{ and thus } Eq_0 = 0. \quad (5)$$

The business processes, including the start up of the organisation, are recorded as transaction  $i = 1, 2, 3, \dots$ . Because A, L, and Eq at point  $i = 0$  are zero, the cumulative changes in A, L, and Eq show the same amounts as the position of these stock values after the recording of transactions:

$$\sum_{i=1}^n \Delta A_i = A_n; \quad \sum_{i=1}^n \Delta L_i = L_n; \quad \sum_{i=1}^n \Delta Eq_i = Eq_n . \quad (6)$$

Equation (7) shows the position of the stock values, as the sum of the changes of these stock values from transaction 1 up to and including transaction n. The form of the balance-sheet formulation is given in the



following equation:

$$\sum_{i=1}^n \Delta A_i = \sum_{i=1}^n \Delta L_i + \sum_{i=1}^n \Delta Eq_i . \quad (7)$$

Equation (7) describes the bookkeeping model as a balance-sheet-model. Because the changes in A, L, and Eq caused by transaction i are added in relation to (4). Besides the input function, (7) also has an output function. The output, which is delivered by (7), is the position of assets and liabilities for the balance sheet up to and including transaction n. The position of equity after the recording of transaction n is found by subtracting assets and liabilities.

Equation (7) is dualistic: input and output are described in only one expression. The output of the balance sheet model in (7) presents the financial position of stock values after the recording of n transactions has been completed. There is a major drawback to this output: the composition of the balance sheet has to include a time dimension, because the balance sheet has to be composed at any moment in time, say ending time ( $T_e$ ). The balance sheet model described in (7) does not allow this, because it is based on n transactions without a time factor. For the time-related balance sheet composition, it is not the number of transactions that is relevant, but the fact that all transactions before the chosen balance sheet moment are included. In other words, transactions that have occurred before this moment must be taken into account. In order to include the time factor in the balance sheet model, every transaction i has to carry a time indicator ( $t_i$ ). This time indicator refers to the moment of the transaction. The balance sheet at  $T_e$  is with (7) the aggregation of all transactions i with a time indicator before or equal to moment  $T_e$  ( $t_i \leq T_e$ ). Reformulation of (7) leads to the following equation:

$$\sum_{i:t_i \leq T_e} \Delta A_i = \sum_{i:t_i \leq T_e} \Delta L_i + \sum_{i:t_i \leq T_e} \Delta Eq_i . \quad (8)$$

Equation (8) is an extension of (7). In (7) moments in time are not taken into account, which is in (8).

### 3.3. Profit and loss statement model

The bookkeeping model to be analysed consists, besides the balance sheet model, of a profit and loss statement model. In the previous section the independent balance sheet model was described; in this section, we concentrate on an independent profit and loss statement model. In section 3.4, the two models are integrated.

In the profit and loss statement model the composition of profit is continuously recorded; the total amount of profit is calculated by balancing.

Profit is described by two components: Income and Expenses. Profit is defined as the difference between income (= I) and expenses (= E):

$$P = I - E \quad (9)$$

Income (I) is defined by the conceptual framework as “increases in economic benefits during the accounting

period in the form of inflows of enhancements of assets or decreases of liabilities that result in increases in equity, other than those relating to contributions from equity participants”. Expenses (E) are defined as “decreases in economic benefits during the accounting period in the form of outflows or depletions of assets or incurrences of liabilities that result in increases in equity, other than those relating to distributions to equity participants” (International Accounting Standards Committee, 1991).

P, I, and E are considered as stock values. They have a position at any point in time. The positions of the stock values P, I, and E up to and including transaction i are defined as  $P_i$ ,  $I_i$ , and  $E_i$ . As a result of transaction i, the stock values change:

$$\Delta P_i = \Delta I_i - \Delta E_i \quad . \quad (10)$$

The changes in the stock values as a result of transaction i are defined as the difference between the position up to and including transaction i and the position up to and including transaction i-1:

$$\Delta P_i = P_i - P_{i-1} \quad . \quad (11)$$

In (10), every transaction causes exactly two changes. Every transaction i leads to a change in I or E. The change in the stock value P is defined by the balancing of the equation. For mathematical reasons, the assumption is made that all stock values change as a result of transaction i. The change can be positive, negative, or equal to zero.

At the start of the organisation (i=0), every stock value equals zero.

$$I_0 = 0; E_0 = 0; \text{ and thus } P_0 = 0 \quad (12)$$

The position of the stock values P, I, and E after the recording of n transactions can again be written as the cumulative mutations in the stock values from transaction 1 up to and including transaction n:

$$\sum_{i=1}^n \Delta P_i = P_n \quad ; \quad \sum_{i=1}^n \Delta I_i = I_n \quad ; \quad \sum_{i=1}^n \Delta E_i = E_n \quad . \quad (13)$$

Equation (13) describes the position of the stock values P, I, and E as the aggregation of the changes as a result of transactions 1 up to and including n. Equation (9) is applicable to these positions. The profit and loss statement can be written as follows:

$$\sum_{i=1}^n \Delta E_i + \sum_{i=1}^n \Delta P_i = \sum_{i=1}^n \Delta I_i \quad . \quad (14)$$

The profit and loss statement can be formulated with (14) as a cumulative statement after n transactions. In practice the profit and loss statement has to be a period-related statement in which profit is calculated over a period of time, say from January 1 till December 31. Equation (14) is not able to supply period-related

output because:

- (a) The model works with stock values, which are transaction-related instead of time-related.
- (b) Because of the periodic aspect of the desired output, the *change* in stock values during a *period* should be measured.

To obtain time-related information we propose the following:

- (a) Transaction-related stock values have to be transformed into time-related stock values. This can be achieved by linking a moment to every transaction  $i$ . The profit and loss account can then be formulated as a cumulative statement up to and including moment  $T_e$ . All transactions to which  $t_i \leq T_e$  applies, are included in the composition of the profit and loss statement.
- (b) In order to unambiguously integrate the profit and loss statement model and the balance sheet model, we will not introduce separate period-related flow values, but we will hold on to moment-related stock values as defined before.

Therefore, a period is defined as the interval between two moments; the desired period has the time-interval  $[T_b, T_e]$ . Profit of a certain period will be calculated as the difference of the (cumulative) stock value Profit between two chosen moments:

$T_e$ : end-moment of the profit-calculating period

$T_b$ : begin-moment of the profit-calculating period

$T_e > T_b$ , which means  $T_e$  is later in time than  $T_b$ .

The cumulative position of the stock value  $P$  at the moments  $T_b$  and  $T_e$  can be calculated as follows:

- $P$  at the desired moment  $T_b$ : all changes caused by transactions  $i$  with time-factor  $t_i < T_b$  have to be aggregated; we assume that these are the transactions 1 up to and including  $k-1$ .
- $P$  over the desired period  $[T_b, T_e]$  is calculated by cumulating the change in profit caused by transactions  $k$  up to and including  $n$ . The profit over this period is defined as the difference between the position of profit up to and including transaction  $n$ , and the position of profit up to and including transaction  $k-1$ .

$$\sum_{i=1}^n \Delta P_i - \sum_{i=1}^{k-1} \Delta P_i = \sum_{i=k}^n \Delta P_i \quad (15)$$

Substitution of equation (14) and (15) lead to the following equation:

$$\sum_{i=k}^n \Delta E_i + \sum_{i=k}^n \Delta P_i = \sum_{i=k}^n \Delta I_i \quad (16)$$

The composition of the profit and loss statement over the transactions  $k$  up to and including  $n$  is based on (16). The profit and loss statement has to be drawn up over a desired period of time. When it is taken into account that for every transaction  $i$  the moment of each transaction  $t_i$  is recorded, profit can be measured over any period  $[T_b, T_e]$ , by calculating the difference between the cumulative profit including all transactions from time zero up to and including  $T_e$ , and the cumulative profit including all transactions from time zero to  $T_b$ . The following equation formalises this statement:

$$\sum_{i:t_i \leq T_e} \Delta P_i - \sum_{i:t_i < T_b} \Delta P_i = \sum_{i:T_b \leq t_i \leq T_e} \Delta P_i . \quad (17)$$

Equation (17) can now be reformulated as:

$$\sum_{i:T_b \leq t_i \leq T_e} \Delta E_i + \sum_{i:T_b \leq t_i \leq T_e} \Delta P_i = \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i \quad (18)$$

Equation (18) describes the model of the profit and loss statement as an independent input-output model. Every transaction is entered as a change in the output variables I or E. The change in P is calculated by subtracting E from I. Equation (18) provides a profit and loss statement for every desired period  $[T_b, T_e]$ .

### 3.4. Integrated model

In this section, the balance model of section 3.2, and the profit and loss statement model of section 3.3, will be integrated. The balance sheet model and the profit and loss statement model are represented in different equations. The most elementary equation for the balance sheet model is stated in (4):

$$\Delta A_i = \Delta L_i + \Delta E_{q_i} \text{ or } \Delta A_i - \Delta L_i - \Delta E_{q_i} = 0 .$$

This equation describes the reaction of the stock values of the balance sheet model to every type of transaction. All other equations that describe the balance sheet model are variations on (4).

The most elementary equation for the profit and loss statement model was given in (10):

$$\Delta P_i = \Delta I_i - \Delta E_i .$$

This equation describes the reaction of the stock values of the profit and loss statement model to every kind of transaction. All other equations describing the profit and loss statement model are variations of (10). In (4), equity balances, similarly in (10), profit balances. The balancing stock value of the balance sheet model and the balancing stock value of the profit and loss statement model are equal; this is caused by the fact that we abstract from causes of change in equity other than profit. Furthermore, the amount of change of equity is equal to the amount of change in profit. These properties are revealed in the following equation:

$$\Delta E_{q_i} = \Delta I_i . \quad (19)$$

After substitution of (4), (11) in (19), the most elementary equation of the integrated model is:

$$\Delta A_i - \Delta L_i = \Delta I_i - \Delta E_i . \quad (20)$$

Equation (21) describes how the four stock values react to transaction i. For mathematical reasons we assume that transaction i from the balance sheet model, is the same as transaction i from the profit and loss statement

model, and therefore, also the same as transaction  $i$  from the integrated model. Transaction  $i$  is called the input for (20). Equation (20) does not deliver output in the form of a balance sheet and profit and loss statement; it only reveals the changes in the stock values - in other words, the working of the bookkeeping model.

With regard to the output, in the balance sheet model as well as in the profit and loss statement, the positions of the stock values are formulated as the aggregate of the changes in the stock values. This is reflected in the equations (7) and (14):

$$\sum_{i=1}^n \Delta A_i = \sum_{i=1}^n \Delta L_i + \sum_{i=1}^n \Delta E_{q_i}, \quad \sum_{i=1}^n \Delta E_i + \sum_{i=1}^n \Delta P_i = \sum_{i=1}^n \Delta I_i.$$

Given that the starting positions of the stock values equity and profit are assumed to be zero (see (5) and (12) respectively) we can argue, in analogy to (19):

$$\sum_{i=1}^n \Delta E_{q_i} = \sum_{i=1}^n \Delta P_i. \quad (21)$$

The cumulative changes in equity up to and including transaction  $n$  equal the cumulative changes in profit up to and including transaction  $n$ . Substitution of (7), (14), in (21) leads to the following equation:

$$\sum_{i=1}^n \Delta A_i - \sum_{i=1}^n \Delta L_i = \sum_{i=1}^n \Delta I_i - \sum_{i=1}^n \Delta E_i \quad (22)$$

Equation (22) can be used for input and generating output. All transactions  $i$  from 1 up to and including  $n$  are input, and the positions of the stock values up to and including  $n$  are output for the balance sheet and the profit and loss statement. We should also be able to generate the profit and loss statement for a particular number of transactions  $k$  up to and including  $n$ . This is reflected in equation (16):

$$\sum_{i=k}^n \Delta E_i + \sum_{i=k}^n \Delta P_i = \sum_{i=k}^n \Delta I_i$$

The right hand side of equation (22) can be divided in:

- the position of profit up to and including transaction  $k-1$  ( $= P_{k-1}$ ), and
- profit as aggregation of changes in income and expenses caused by transactions  $k$  up to and including  $n$ .

This division is made clear in (23):

$$\sum_{i=1}^n \Delta I_i - \sum_{i=1}^n \Delta E_i = \sum_{i=1}^{k-1} \Delta I_i - \sum_{i=1}^{k-1} \Delta E_i + \sum_{i=k}^n \Delta I_i - \sum_{i=k}^n \Delta E_i, \quad (23)$$

where

$$\sum_{i=1}^{k-1} \Delta I_i - \sum_{i=1}^{k-1} \Delta E_i = P_{k-1} . \quad (24)$$

Substitution of (16), (22), (23) in (24) leads to:

$$\sum_{i=1}^n \Delta A_i - \sum_{i=1}^n \Delta L_i = P_{k-1} + \sum_{i=k}^n \Delta I_i - \sum_{i=k}^n \Delta E_i . \quad (25)$$

Equation (25) is applicable for input and for generating output. Input are all transactions  $i$  from 1 up to and including  $n$ . With the left hand side of the equation, we can calculate the positions of the stock values up to and including transaction  $n$  of the balance sheet. The extent of equity can be found in the right hand side of the equation. The last term of the right hand side of the equation gives the positions of stock values for the profit and loss statement from transaction  $k$  up to and including  $n$ . To obtain the balance sheet and profit and loss statement for a desired period, we introduce the time related index  $t_i$ :

$$\sum_{i:t_i \leq T_e} \Delta A_i = \sum_{i:t_i \leq T_e} \Delta L_i + \sum_{i:t_i \leq T_e} \Delta E_{q_i} .$$

In the profit and loss statement model, every transaction  $i$  also includes the moment  $t_i$ . Equation (18) also indicates the moment  $t_i$  for transaction  $i$ :

$$\sum_{i:T_b \leq t_i \leq T_e} \Delta E_i + \sum_{i:T_b \leq t_i \leq T_e} \Delta P_i = \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i$$

Because of the fact that transaction  $i$  is the same transaction for the balance sheet model, and the profit and loss statement model, the accompanying moment  $t_i$  is the same for both models too. The integrated model, simultaneously supplying the balance sheet at the desired moment  $T_e$  and the profit and loss statement for the desired period  $[T_b, T_e]$ , is realised based on the following property:

$$\sum_{i:t_i \leq T_e} \Delta E_{q_i} = \sum_{i:t_i \leq T_e} \Delta P_i . \quad (26)$$

Substitution of (8), (17), (18) in (26) produces the following equation:

$$\sum_{i:t_i \leq T_e} \Delta A_i - \sum_{i:t_i \leq T_e} \Delta L_i = \sum_{i:t_i \leq T_e} \Delta I_i - \sum_{i:t_i \leq T_e} \Delta E_i \quad (27)$$

Equation (26) can not generate output for a particular period  $[T_b, T_e]$ . The right hand side of the equation has to be written in two parts:

- Cumulative changes in  $P$  from starting point to the beginning of the desired period for which we want to compose the profit and loss statement.
- Cumulative change in  $P$ , divided into  $I$  and  $E$  over the desired period  $[T_b, T_e]$ .

$$\sum_{i:t_i \leq T_e} \Delta I_i - \sum_{i:t_i \leq T_e} \Delta E_i = \sum_{i:t_i < T_b} \Delta I_i - \sum_{i:t_i < T_b} \Delta E_i + \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i - \sum_{i:T_b \leq t_i \leq T_e} \Delta E_i ,$$

(28)

where

$$\sum_{i:t_i < T_b} \Delta I_i - \sum_{i:t_i < T_b} \Delta E_i = P_{[0, T_b)} .$$

(29)

Combining (27) and (28) leads to:

$$\sum_{i:t_i \leq T_e} \Delta A_i - \sum_{i:t_i \leq T_e} \Delta L_i = \sum_{i:t_i < T_b} \Delta I_i - \sum_{i:t_i < T_b} \Delta E_i + \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i - \sum_{i:T_b \leq t_i \leq T_e} \Delta E_i ,$$

(30)

and after substitution of equation (29) in (30):

$$\sum_{i:t_i \leq T_e} \Delta A_i - \sum_{i:t_i \leq T_e} \Delta L_i = P_{[0, T_b)} + \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i - \sum_{i:T_b \leq t_i \leq T_e} \Delta E_i .$$

(31)

Equation (31) can be used for entering input, and generating output. Input are the transactions  $i$  with accompanying moment  $t_i$ . The periods  $[0, T_e]$  and  $[T_b, T_e]$  represent the timeframe over which input and output are taken into account.

With reference to the output, both (31) and (27) describe the balance sheet in the left hand side of the equation. The left hand side gives the composition of equity on the desired balance sheet at moment  $T_e$ . The extent of equity equals the amount of the right hand side of the equation at moment  $T_e$ . The difference between (31) and (27) lies in the right hand side of the equation. In the right hand side of (31), the profit and loss statement can be found as composed over the desired period  $[T_b, T_e]$ . If moment  $T_b$  belongs to transaction 1, (31) coincides with (27). The extent of the right hand side of the equation also equals the extent of the cumulative profit up to and including moment  $T_e$ . In order to compose the profit and loss statement over the desired period  $[T_b, T_e]$ , the extent of the cumulative profit has to be split up. Cumulative profit can be divided into the cumulative changes in profit as position at moment  $T_b$  and the cumulative changes in profit during period  $T_b$  up to and including  $T_e$ , split up into changes of  $I$  and  $E$ .

In this section, integration is achieved based on two autonomous models: balance sheet model and profit and loss statement model. The balance sheet model focuses on the recording of the composition of equity. The extent of equity is calculated by balancing.

The profit and loss statement model focuses on the recording of the composition of profit. The extent of profit is calculated by balancing. Both models have a recording based on single-entry bookkeeping.

By integrating the two models in the integrated model, we find the recording on the composition of equity and the composition of profit or the extent of equity. The integrated model thus has a recording based on double-entry bookkeeping.

### *3.5. Example of an employment agency*

To illustrate the working of the integrated model, we have chosen an employment agency. This organisation's main activity is lending out personnel to other organisations. Activities start with transaction 1. Before transaction 1, there are no assets ( $A_0 = 0$ ), liabilities ( $L_0 = 0$ ), or equity ( $Eq_0 = 0$ ). In the deployment of temporary staff, the following procedure applies:

The employee will be stationed with a certain firm. During a certain period, this firm pays a fee (allowance) for the hired employee. The employment agency itself pays wages to the employee over the same period. There is a gross margin between the received allowances with regard to the employee and the wages paid. All transactions are to be considered cash transactions. The business processes of the employment agency can be reproduced in the value cycle of Figure 1:



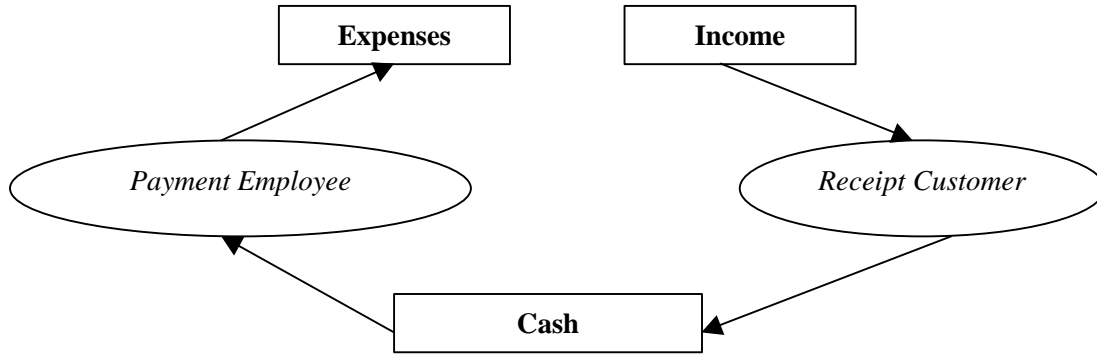


Figure 1: Value cycle of an employment agency

The value cycle explains how stock values are transformed due to business processes. The employment agency can be seen as a service organisation with no physical flow of goods. The fact that all transactions are on a cash basis implies a transformation Assets into Assets.

The employment agency only processes assets and no liabilities. Equation (31), describing the bookkeeping model, is applicable to the employment agency:

$$\sum_{i:t_i \leq T_e} \Delta A_i - \sum_{i:t_i \leq T_e} \Delta L_i = P_{[0, T_b)} + \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i - \sum_{i:T_b \leq t_i \leq T_e} \Delta E_i .$$

The business processes of the employment agency can be transformed into two relevant transactions:

- Receipt of liquidity (increase of A) as an allowance for the activities of the hired employee at a certain moment;
- Payment of liquidity (decrease of A) as a salary to the hired employee at a certain moment.

As a consequence of (a), the left hand side of (31) increases. Since (31) is always in balance, this means that the right hand side also has to increase. In other words, P increases as a result of this transaction. This increase in P can be classified as an increase in the stock value I at the moment of the transaction (there is no change in the stock value E). After this transaction, it is possible to compose the balance sheet and the profit and loss statement with (31). The balance sheet can be immediately composed with the help of the left hand side of the equation. The profit and loss statement is a period-related statement. The beginning moment Tb and the ending moment Te of the period of the profit and loss statement have to be indicated. Then the profit and loss statement regarding this period can be composed with the help of the right hand side of (31). Transaction (b) causes a decrease in Liquidity and also a decrease in A. As a result, P will decrease at the moment of the transaction. This decrease in P is caused by E. The equation is also in balance with this transaction because the change in A equals the change in E. After this transaction, the balance sheet and the profit and loss statement can be composed. The fact that P is calculated over a desired period [Tb, Te], so that the beginning moment (Tb) and the ending moment (Te) of the period have to be indicated, is now also taken into account.

### § 3.6 Extension of the model

The model described before is a bookkeeping model that delivers the required output to a high degree of abstraction. The model can be extended by abandoning the assumption that equity will only change as a result of profit.

The extension in this section with regard to the preceding sections is that equity not only can change by P, but also by Withdrawals (W) and Contributed Capital (CC). In the extended model, W and CC are also defined as stock value X, which means that the position of W and CC up to and including transaction i is presented as  $CC_i$ , i.e. the position of CC, and  $W_i$ , i.e. the position of W.

CC and W can be built into the integrated model as follows: change in Eq is no longer equal to the change in P, but the change in Eq will be interpreted by a change in P, W, or CC. Therefore the following equation applies:

$$\Delta P_i + \Delta CC_i - \Delta W_i = \Delta E_{q_i} . \quad (32)$$

In the foregoing sections, it has been assumed that at the starting moment of the firm, all stock values equal zero. This also applies to this extension:

$$CC_0 = 0 ; W_0 = 0 . \quad (33)$$

After substitution of (31) and (32), the following equation for the bookkeeping model including contributed capital and withdrawals can be given:

$$\sum_{i:t_i \leq T_e} \Delta A_i + \sum_{i:t_i \leq T_e} \Delta L_i = \sum_{i:t_i \leq T_e} \Delta CC_i - \sum_{i:t_i \leq T_e} \Delta W_i + P_{[0, T_b)} + \sum_{i:T_b \leq t_i \leq T_e} \Delta I_i - \sum_{i:T_b \leq t_i \leq T_e} \Delta E_i . \quad (34)$$

The left part of the hand side of (34) contains the composition of Eq up to and including transaction n with the moments up to and including the moment of the balance sheet  $T_e$ . In the right part of the equation, the total amount of Eq can be found using the following components:

- cumulative change in CC and W up to and including transaction n and therefore also the position of stock values at moment  $T_e$ .
- cumulative P up to and including transaction n and up to and including moment  $T_b$ .
- cumulative P over the period  $[T_b, T_e]$ , broken down in I and E.

## 4. Different levels of abstraction of the conceptual bookkeeping model

### § 4.1 Introduction

In the previous section, an analysis of the bookkeeping model was given. By means of equations, an independent balance sheet model and an independent profit and loss statement model were described. Both

models were combined into an integrated model. The starting-point for each model was a definition equation. This section aims at the different levels of specification as present in section 3. Furthermore, some more levels are added to the model formulated before. The different levels of specification correspond to the different levels of abstraction of the conceptual bookkeeping model. The model at the highest level of abstraction shows the functioning of the system; the version with the least abstraction tends towards a practical bookkeeping system. The different levels of the conceptual model are explained in the following sections. The entry level is called level zero and corresponds with the basic functioning, where the model does not deliver financial output. This level is the most abstract level. Every higher level (1,2,3 etc.) is less abstract because:

- every higher level contains a new element of input
- there is wider and more specific output.

Every level will, if and as far as possible, be described with an equation derived for the integrated model of Section 3.

#### § 4.2 *Level zero*

Level 0 describes the functioning of the most general model. This level, from which all other levels are derived, has no autonomous output. This is the reason why this level is called “zero”, all other levels do have a balance sheet and profit and loss statement as output.

The transactions, formalised in a transaction code, are the input of this level. Level zero describes how, as a consequence of transactions, the stock values in equation (20) will change. In other words: level zero pictures the reaction of the equation for every transaction:  $\Delta A_i - \Delta L_i = \Delta I_i - \Delta E_i$  .

In an organisation, different kinds of transactions exist which cause different changes in (20). Every transaction leads to two changes in the equation containing four stock values. The change either is more than, less than, or equal to zero. The two non-zero mutations are equal in volume. This can be explained from the perspective of the circulation of values, where one value is increasing and the other value is decreasing (see Figure 1). Compare also the double-entry system, every transaction has a debit amount equal to a credit amount (see section 2.3). Equation (20) therefore, as a consequence of transactions, can react in  $4^2 = 16$  ways. These 16 technically possible reaction-patterns of (20) are numbered with a transaction code, which is shown in Table 1.

Table 1: Technically possible reaction-patterns in a bookkeeping-system

Transaction code	$\Delta A > 0$	$\Delta A < 0$	$\Delta L > 0$	$\Delta L < 0$	$\Delta I > 0$	$\Delta I < 0$	$\Delta E > 0$	$\Delta E < 0$
1	x	x						
2	x		x					
3	x				x			
4	x							x
5		x		x				
6		x				x		
7		x					X	
8			x	x				
9			x			x		
10			x				X	
11				x	x			
12				x				x
13					x	x		
14					x		x	
15						x		x
16							x	x

Level zero depicts the functioning of the bookkeeping model. Transactions are input for the bookkeeping model. Level zero indicates, by means of the transaction code, which stock values change as a result of a certain kind of transaction. The stock values, which actually change, are marked in Table 1. Stock values, which do not change as a result of a transaction, are depicted as empty cells in Table 1 (Formally, their change is equal to zero).

In the bookkeeping system, a positive change, will be indicated by +1, a negative change by −1, and a change equal to zero by 0. It is possible to check the correctness of the indicated changes: the sum of each row has to be zero. Changes in stock values are caused by transactions of which there are many types. All these transactions can be linked to a row in Table 1. Some reactions are more usual than others. Reactions 13 up to and including 16 are technically possible, but rare in practice. The reason for this is that in a double-entry system, equity is recorded in composition and extent. As result of reactions 13 up to and including 16, equity changes neither in composition nor in extent.

Most transactions have to be regarded as ex post transactions, since the business processes reflected by transactions lie in the past. The transactions budgeted for business processes in the future have an ex ante character. These ex ante transactions can also be used as input for equation (20) and have the reaction-pattern shown in Table 1.

Transactions and reaction-patterns can be clearly expressed in a value cycle. As an example, we take the value cycle of a trading company; see Figure 2. The value cycle shows the primary process and various accompanying stock values. With respect to the different kinds of transactions in this paper, we choose to limit the transactions to the primary transactions of a trading company. Besides these primary transactions, a commercial enterprise conducts other kinds of transactions supporting the primary process. Each type of business has different varieties of primary and supportive transactions. But for every type of transaction in any organisation, the change in equation (20) can be determined, as described before, by increasing (decreasing) the value cycle. Every type of transaction has one possible reaction, but several kinds of transactions can have the same reaction-pattern.

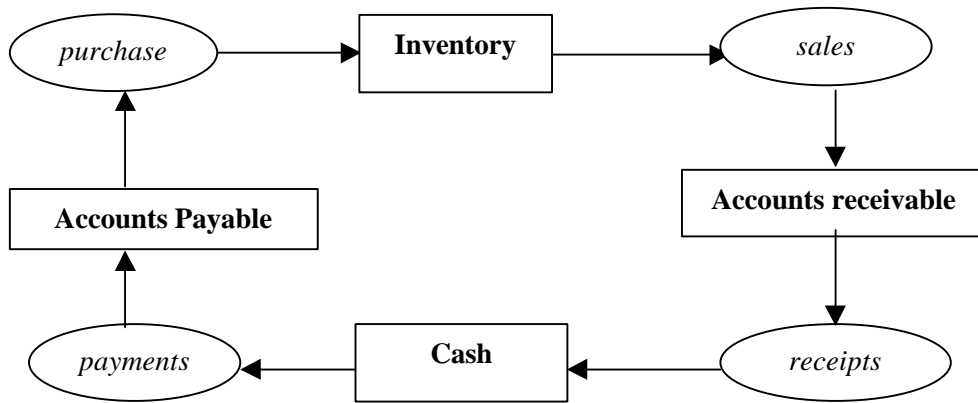


Figure 2: Value cycle in a trading company (Starreveld, 1993, p. 99)

In Figure 2, the rectangles show the stock values of the organisation. The ovals depict the primary transactions or processes. As result of every transaction, the foregoing stock value decreases and the next stock value increases. This is the reason that every transaction causes two changes in stock values. This value cycle can be shown in terms of equation (21) as follows:

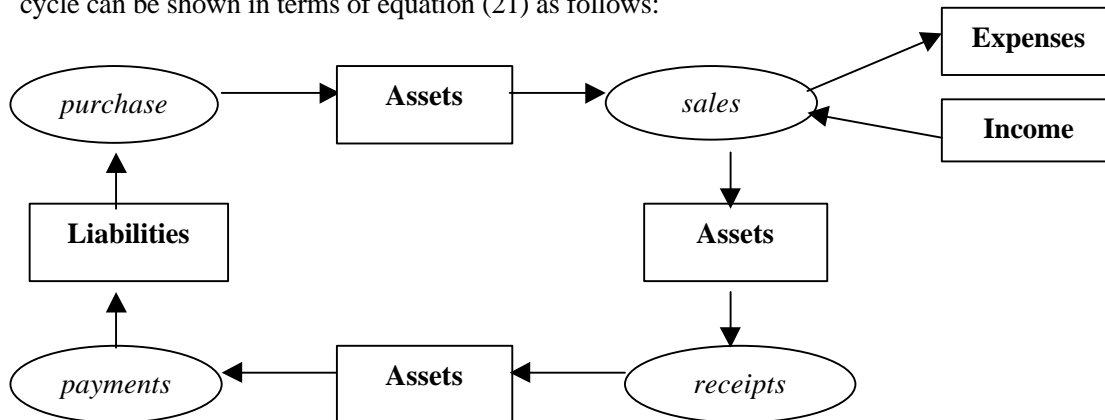


Figure 3: Value cycle in terms of (20)

Figure 2 depicts the value cycle with the stock values of (20). The purchase transaction causes an increase in assets and a decrease in liabilities by an equal amount. As liabilities have a negative sign in (20), they show a negative decrease, i.e. an increase. The purchase transaction is indicated by transaction code 2 in Table 1. The sales transaction causes an increase in value: the gross margin of sales transaction. To show this rise in value, we divide the sales transaction into two sub-transactions. As a result of the first sub-transaction, assets decrease and expenses increase. This sub-transaction is indicated in Table 1 by transaction code 7. As a result of the second sub-transaction, assets increase and income decreases by a higher amount. But Income has a negative sign, so Income shows a negative decrease, i.e. an increase. This sub-transaction is indicated in Table 1 by transaction code 3.

The transaction receipt causes an increase and a decrease in Assets at the same moment. This transaction is indicated in Table 1 by transaction code 1.

The transaction payment decreases Assets and causes a negative increase (=decrease) in Liabilities. This transaction is indicated in Table 1 by transaction code 5.

The reactions of the primary processes of the trading company are indicated by transaction codes 1,2,3,5 and 7. As mentioned above, the reactions 13 up to and including 16 hardly ever occur. The other transaction codes indicate the effects of supporting processes.

The only input of basic level zero is:

- transaction code.

The output of level zero has no practical use, because it is not measured in money. At this level, we focus on the working of the model without regard to the output. As such, this level serves as a basis for other levels. Next we reduce the level of abstraction.

### § 4.3 Level 1

Because level zero does not generate autonomous output, level 1 is the first level generating output in the sense of cumulative positions up to and including transaction n. The equation describing this is (22):

$$\sum_{i=1}^n \Delta A_i - \sum_{i=1}^n \Delta L_i = \sum_{i=1}^n \Delta I_i - \sum_{i=1}^n \Delta E_i .$$

In (22), changes in stock values caused by transactions i are summed for transaction 1,...,n. Transactions are not numbered. At level zero, the extent of the change was not important, at this level the extent of the change does matter. As the output will be in money, the input at this level has to be the transaction amount. The input at this level consists of:

- transaction code (level 0);
- transaction amount (level 1).

The output of this level are the positions of assets and liabilities up to and including transaction n, i.e. the balance sheet, and the positions of income and expenses up to and including transaction n, i.e. the profit and loss statement. It is not possible to compose a profit and loss statement over a desired number of transactions, because at this level the transactions are not numbered yet. An individual transaction cannot be distinguished after being entered. When, at this level, budgeted transactions are used as input the output will be a projected balance sheet and a projected profit and loss statement. Level 1 is an extension of level zero by introducing transaction amounts.

### § 4.4 Level 2

Level 1 cannot produce a profit and loss statement over a desired number of transactions, because the transactions are not numbered. By giving every transaction a unique number, one may compose the profit and loss statement over, for example, a transaction k up to and including n. Level 2 can be described by equation (25):

$$\sum_{i=1}^n \Delta A_i - \sum_{i=1}^n \Delta L_i = P_{k-1} + \sum_{i=k}^n \Delta I_i - \sum_{i=k}^n \Delta E_i$$

The difference between (22) of level 1 and (25) of level 2 is that, besides the amount, also an unique transaction number is entered. With the numbered transactions, it is possible to make a balance sheet and a profit and loss statement over a desired number of transactions. Because the changes in the extent of wealth are the most interesting, the profit and loss statement is divided into two parts in (25): cumulative profit up to and including transaction k-1 and the profit and loss statement with the exploratory values Income and Expenses. With respect to level 1, at this level, we can compose a profit and loss statement over a desired number of transactions. Input of level 2 consists of:

- transaction code (level 0);
- transaction amount (level 1);
- transaction number (level 2).

When transactions are identified by unique transaction numbers, an audit trail to identify each transaction is possible. Furthermore, at this level, it is possible to compose the (possibly projected) balance sheet after a desired number of transactions and the (possibly projected) profit and loss statement over a desired number of transactions: transaction k up to and including n. We are, however, not able to select a particular point in time, or time period to formulate these statements.

#### § 4.5 Level 3

The balance sheet and the profit and loss statement are not composed based on a number of transactions, but based on a particular moment and a particular period respectively. We can introduce time by linking the moment of entry ( $t_i$ ) to every transaction  $i$ .  $t_i$  is formulated in terms of calendar time: a date and, if desired, the hour/minute/second on the day in question. This level is described in equation (31):

$$\sum_{i: t_i \leq T_e} \Delta A_i - \sum_{i: t_i \leq T_e} \Delta L_i = P_{[0, T_b)} + \sum_{i: T_b \leq t_i \leq T_e} \Delta I_i - \sum_{i: T_b \leq t_i \leq T_e} \Delta E_i$$

Since the moments at which stock values change are recorded, a balance sheet can be drawn up at every desired moment by taking into account all transactions prior to the desired moment  $t_i < T_e$ , i.e. the period  $[0, T_e]$ . This period can divide in two periods:  $[0, T_b)$  en  $[T_b, T_e]$ . The separation of these periods is less interesting for the balance sheet components A and L. The composition of wealth, defined by A and L, at a desired moment is interesting. Because the changes in wealth over a desired period are also interesting, there is a separation of periods for I and E. All transactions during  $[0, T_b)$  and  $[T_b, T_e]$  are summed for cumulative profit at  $T_e$  and the profit and loss statement over  $[T_b, T_e]$ . The input at this level has a moment  $t_i$  belonging to transaction  $i$ , as a supplement to level 2. The input belonging to this level, therefore, is:

- transaction code (level 0);
- transaction amount (level 1);
- transaction number (level 2);
- transaction date (level 3).

Output is the (possibly projected) balance sheet over period  $[0, T_e]$ , and the (possibly projected) profit and loss statement over any desired period  $[T_b, T_e]$ .  $T_b$  and  $T_e$  normally are calendar-days (possibly including the time of the day). The model includes transactions 1 up to and including n for the balance sheet and

transaction  $k$  up to and including  $n$  for the profit and loss statement. Neither balance sheet nor profit and loss statement are drawn up in detail: the balance sheet consists of three elements: total assets, total liabilities, and total equity. The profit and loss statement also consists of three total amounts: income, expenses, and profit. Level 3 is in accordance with the integrated model developed in section 3. Up to and including this level, a bookkeeping model can work and deliver useful output, but in aggregated terms, which do not fulfil the requirements of legislation and regulation.

#### § 4.6 Level 4

The extension of level 3 lies in the specification of stock values. Legislation and regulations lay down requirements of form and content regarding balance sheet and profit and loss statement. In practice, different categories of A, L, I, and E are distinguished. For example, Asset category 1 stands for buildings, Asset category 2 stands for Inventory etc. For mathematical reasons, we assume that  $j$  has values from 1 up to and including  $m$ , and that, within every stock value, an equal number of specifications is used, namely  $m$ .

Level 4 is distinct from the previous levels by the degree of detail; equation (31) has been extended as follows:

$$\sum_{j=1:i:t_i \leq T_e}^m \sum \Delta A_i^j - \sum_{j=1:i:t_i \leq T_e}^m \sum \Delta L_i^j = P_{[0,T_b)} + \sum_{j=1:i:T_b \leq t_i \leq T_e}^m \sum \Delta I_i^j - \sum_{j=1:i:T_b \leq t_i \leq T_e}^m \sum \Delta E_i^j \quad (35)$$

In (35), every stock value of (31) is specified by an identification code  $j$ , which points out which category of the stock values A, L, I, or E is concerned. The input for this level is:

- transaction code (level 0);
- transaction amount (level 1);
- transaction number (level 2);
- transaction date (level 3);
- identification code  $j$  for every category of stock values.

The output at this level is a (possibly projected) balance sheet over the desired period  $[0, T_e]$  and a (possibly projected) profit and loss statement over a desired period  $[T_b, T_e]$ , complying with the requirements of form for financial statements. Changes in stock values are also aggregated over  $j$ . Therefore, the output might possibly not be in stock values  $j$ , but in the aggregate of all  $j$  by stock value. This aggregation takes place in order to achieve mathematical correctness of (35). The presumption is that input will be retained. At this level compared to level 3, extra output can be generated:

- changes and positions of specific stock values;
- more insight into the composition of equity;
- insight into the explanatory components of profit: income, and expenses are in more detail specified, which can provide insight into, for example, the specific type of costs.

#### § 4.7 Level 5



In comparison with level 4, level 5 offers an additional specification of stock values. This additional specification is represented by the supplementary variable  $s$ , an individual class standing, for example, for any account receivable or payable, for any kind of building, for the costs of a cost center, etc.. Equation (35) at this level has to be expanded as follows:

$$\sum_{s=1}^r \sum_{j=1:t_i \leq T_e}^m \Delta A_i^{j,s} - \sum_{s=1}^r \sum_{j=1:t_i \leq T_e}^m \Delta L_i^{j,s} = P_{[0,T_b)} + \sum_{s=1}^r \sum_{j=1:T_b \leq t_i \leq T_e}^m \Delta I_i^{j,s} - \sum_{s=1}^r \sum_{j=1:T_b \leq t_i \leq T_e}^m \Delta E_i^{j,s} . \quad (36)$$

In equation (36) identification code  $s$  means a further specification of identifier  $j$ . Identification code  $s$  has a range from 1 up to and including  $r$ . Input for this level has to be:

- transaction code (level 0);
- transaction amount (level 1);
- transaction number (level 2);
- transaction date (level 3);
- identification code  $j$  for every category of stock values (level 4);
- identification code  $s$  for every sub-category of a category of stock values (level 5).

In (36), all identifications  $s$  are aggregated because these are not significant for the (possibly projected) output of the balance sheet and the profit and loss statement. The specifications are retained (compare level 4, identification  $j$ ). The supplementary output of this extra specification lies in:

- Accounts receivable or payable for individual debtors and creditors: in other words, the subsidiary ledgers;
- Balance received/paid at any moment;
- Overview of all changes of every specification of stock values in time;
- Costs classified to cost centers.

At this level, there is more detailed output than required by legislation and regulation. This extra output is important for the management of an organization. Thus, the output is at this level also management information. The bookkeeping model is become also a management information system.

Higher detailed input can possibly lessen the amount of input. By means of the specification character of the identification code, suppositions can be made. For example: the input of a certain transaction code causes  $A$  and  $L$  to change on the basis of level 0. By creating an individual sub-category of  $L$ , say 'construction' company, marked by identification code  $s$  (with category of  $L$ , accounts payable, marked by identification code  $j$ ), we can assume that this transaction always deals with the purchase of a tangible fixed Asset (building). The identification code  $j$  can then be omitted.

#### § 4.8 Level 6

The extension of level 6 compared to level 5 is that the input for a non-monetary account is no longer an amount of money, but the quantity and price behind it. Quantity is defined as unit type: e.g. pieces, kilograms, hours, etc, which all have a corresponding unit price. Up to and including level 5, the change in stock values is recorded in money. At this level, the changes in every stock value  $X$  will be written as a change in quantity,  $\Delta x_i$ , multiplied by the price of the transaction,  $p_i$ :

$$\Delta X_i = p_{Xi} \Delta x_i \quad (37)$$

For monetary accounts quantity equals the amount of money. For these accounts, we assume the quantity to be equal to the amount of money and the price to be equal to 1.

Expansion of (36) with quantity and price as defined by (37) leads to the following equation:

$$\begin{aligned} \sum_{s=1}^r \sum_{j=1:t_i \leq Te}^m \sum p_{Ai} \Delta a_i^{j,s} - \sum_{s=1}^r \sum_{j=1:t_i \leq Te}^m \sum p_{Li} \Delta l_i^{j,s} = \\ P_{(0,Tb)} + \sum_{s=1}^r \sum_{j=1:t_i: Tb \leq t_i \leq Te}^m \sum p_{Fi} \Delta f_i^{j,s} - \sum_{s=1}^r \sum_{j=1:t_i: Tb \leq t_i \leq Te}^m \sum p_{Ei} \Delta e_i^{j,s} \end{aligned} \quad (38)$$

Equation (38) is equal to equation (36). Because quantity and price are applied in (38), it is made clear that these variables are to be considered input variables at this level. As input, therefore, the following are effective:

- transaction code (level 0);
- transaction number (level 2);
- transaction date (level 3);
- identification code j (level 4);
- identification code s (level 5);

and transaction amount (level 1) is replaced by:

- quantity (level 6);
- price (level 6).

All inputs will be retained in their original form. Additional output at this level is:

- (possibly projected) statements of quantitative inventory levels (in quantity);
- information on efficiency with regard to usage (pieces, kilograms, hours, etc.);
- information about stock of machinery, stock of cars etc..

At this level, the extra output is quantitative information. This information is meant to control the processes in the organization, and therefore management information. The bookkeeping model at this level is also a management information system.

## § 5 Conclusion

There exists a discrepancy between the conceptual bookkeeping model in the bookkeeping/accounting literature and the bookkeeping model used in practice. The conceptual bookkeeping model in the literature is based on the manual bookkeeping system using manual techniques. Modern bookkeeping systems are, however, computerized.

In this paper, we have attempted to develop a new bookkeeping model not based on traditional manual techniques. The starting-point of the model is the accounting equation applied as bookkeeping theory in the Anglo-Saxon literature. The model is sophisticated and is described by a system of equations. The new model distinguishes different levels of abstraction. From these different levels, we analyze bookkeeping techniques.

The most abstract level only describes the working of the bookkeeping model, based on the transaction and delivers the reaction of the stock values in equation (21). This level 0 delivers no directly usable information. For that reason, this level is called level zero. The following levels have each one extra input element and every level delivers more, or more detailed, information. The equation at level 3 describes a good working bookkeeping model, but the output does not comply with regulation and legislation. Level 4 delivers more detailed information and, as a result, can deliver output, which fulfils legislation and regulation. Levels 5 and 6 are the least abstract ones and distribute, based on a number of input elements, diverse very detailed financial and also quantitative output. Because level 6 has quantity as extra input element, it is possible to generate more new statements, especially quantitative ones. Every higher level than level 6 will take into account non-financial input and output elements. The model we discussed last, therefore, is not the highest possible and least abstract model.

The first, more abstract, levels focus mainly on the financial aspect of the transaction. Therefore, the output belonging to these levels is mainly financial. The model at these levels can be characterized as a financial information system. At the later, least abstract levels, the input contains more detailed and quantitative characteristics of the transaction. As a result, the output at the later levels is more aimed at the management. The model has become closer to a management information system.

With this new conceptual bookkeeping model, we have achieved an adaptation of bookkeeping or the accounting literature to the state-of-art technology of today's bookkeeping practice. It is interesting to investigate whether existing bookkeeping models in software are based on the conceptual bookkeeping model developed in this paper and what the correspondence is between these models. Furthermore, it seems interesting to investigate whether this model is applicable as a new educational bookkeeping method.

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